

*James M. Buchanan*

## **An Ambiguity in Sen's Alleged Proof of the Impossibility of a Pareto Libertarian\***

*Abstract:* 'Minimal liberalism', in Sen's strict definition, is impossible, because any 'social state', once chosen, freezes all of its components, thereby removing any prospect of further assignment of choice-making authority.

My purpose here is to identify a basic ambiguity in A. K. Sen's initial note in the *Journal of Political Economy* (Sen 1970), an ambiguity that remains in the later elaboration of his thesis (1976), and one that has somewhat surprisingly not been specifically discussed in any of the several critical reactions that have been variously published.<sup>1</sup> In his note, Sen claimed to prove that the Pareto principle conflicts with the rule or principle of Minimal Liberalism, which he has rechristened as Minimal Libertarianism in his 1976 paper. I shall show that Sen's rule of minimal libertarianism is self-contradictory. When the rule

---

\* *Prefatory Note (April 1995):* I appreciate the decision of the Editors, through the intercession of Professor Hartmut Kliemt, in offering me the opportunity to publish belatedly this note, which was written in September-October 1976 for seminar presentation at the Center for Study of Public Choice, then at Virginia Polytechnic Institute and State University in Blacksburg, Virginia. At that time I made no attempt to publish the note because, in the seminar presentation itself, the argument was so severely criticized by my colleagues that I lost confidence and concluded that, somehow, I must have been confused. Then or now, I have been unable to understand and appreciate the source of the criticisms. Only one xerox copy of the note was circulated beyond my office, mailed to a welfare economist in England, shortly after the seminar. Hence, my surprise, three years later when I received an inquiry from Professor Amartya Sen asking me where the note was published. Sen indicated that he wanted to use my note as a reference for students, and seemed surprised at its fate. He has, since that time, made several references to the note. I should also indicate here that I have made no concerted effort to examine the post-1976 literature on the Pareto-liberal paradox in order to bring the argument 'up to date' as of 1995.

<sup>1</sup> For a complete listing of both the criticisms and Sen's responses, see Sen 1976. The two critics who have come closest to raising the central point of this note are Peter Bernholz 1974 and Christian Seidl 1975. As I shall note later, however, neither Bernholz nor Seidl directly attack Sen's argument at its most vulnerable point, with the result that neither criticism has been effective in dislodging Sen's adherence to his main theorem.

or principle is reformulated so as to make it consistent with what seems to have been Sen's intent, the alleged conflict with the Pareto principle can be shown to be nothing more than the familiar inefficiency of independent adjustment or Nash equilibria under conditions of Pareto-relevant externalities.

The alternatives for comparison in Sen's analysis are "social states", with "each social state being a complete description of society including every individual's position in it" (Sen 1970, 152). His Condition *L* (Liberalism) is defined as follows: "For each individual *i*, there is at least one pair of alternatives, say  $(x, y)$  such that if this individual prefers  $x$  to  $y$ , then society should prefer  $x$  to  $y$ ; and if this individual prefers  $y$  to  $x$ , then the society should prefer  $y$  to  $x$ ." (Sen 1970, 153) This more restrictive condition is then replaced by the lesser requirement that at least two persons in the society, rather than every person, must each be decisive over one pair of alternatives. This difference in defining the rule of liberalism or libertarianism is not relevant to my argument, and it may be neglected in what follows.

Having defined the rule of liberalism, Sen then proceeds to prove that it is inconsistent with Condition *P* (the Pareto principle) which he defines in the standard manner. His alleged inconsistency here is wholly irrelevant, however, because the principle or rule of liberalism, as defined, is self-contradictory. This stems from the elementary fact that not more than one person could ever be assigned so much as a single choice over alternatives, provided that these alternatives are defined as "complete descriptions of society including every individual's position in it". This fact follows from the mutual exclusiveness of the alternatives themselves. Hence, the assignment of decisiveness to a single person necessarily precludes a similar assignment to any other person in the society.

To illustrate this point, let us introduce a simple two-person example, where complete social states can be easily described. Person 1 and Person 2 may be clean-shaven or bearded. All other aspects of their environment, including behavior, are arbitrarily fixed. There are only four possible social states, as follows:

- (*x*) Person 1 clean-shaven; Person 2 clean-shaven;
- (*y*) Person 1 bearded; Person 2 clean-shaven;
- (*z*) Person 1 clean-shaven; Person 2 bearded;
- (*w*) Person 1 bearded; Person 2 bearded.

Let us now define these four mutually exclusive and distinct alternatives as  $x$ ,  $y$ ,  $z$  and  $w$ , the notation introduced in Sen's formal proof of his theorem. Sen proceeds as follows: "Let 1 prefer  $x$  to  $y$ , and 2 prefer  $z$  to  $w$ . And let everyone in the community including 1 and 2 prefer  $w$  to  $x$  and  $y$  to  $z$ . There is no contradiction for 1 and 2, for 1 simply prefers  $w$  to  $x$ ,  $x$  to  $y$  and  $y$  to  $z$ , while 2 prefers  $y$  to  $z$ ,  $z$  to  $w$  and  $w$  to  $x$ . But by Condition  $L^*$  society should

prefer  $x$  to  $y$  and  $z$  to  $w$ , while, by the Pareto principle, society must prefer  $w$  to  $x$  and  $y$  to  $z$ ." (Sen 1979, 154, italics supplied)

The contradiction lies in the inference that Person 1 can somehow be assigned the choice between  $x$  and  $y$  while, *at the same time*, Person 2 is assigned the choice between  $z$  and  $w$ . But it should be evident from the example that the choice of either  $x$  or  $y$  by Person 1 will, in fact, preclude any exercise of choice between  $z$  and  $w$  by Person 2, and vice versa. Since the four social states are mutually exclusive, by definition, it follows that not more than one person can be allowed to be decisive, either between any two of the alternatives or among those in the whole set. The assignment of decisiveness to Person 1 as between the pair  $x$  and  $y$  rules out  $z$  and  $w$  as alternatives between which Person 2 could possibly be allowed to choose. Such an assignment of decisiveness is equivalent to an initial specification of 'property rights'.

In accordance with the preference rankings used in Sen's proof, neither  $x$  nor  $z$  is Pareto-optimal. Either one of these two possible initially chosen points (depending on whether Person 1 or Person 2 is assigned the right of choice) is Pareto-dominated, the first by  $w$ , the second by  $y$ . If Person 1 is, for some arbitrary reason, assigned the right to choose only between  $x$  and  $y$ , but any move from either one of these positions is allowed upon unanimous consent, then society will shift to position  $w$ , which is Pareto-optimal. Similarly, the assignment of the initial right to select between  $z$  and  $w$  to Person 2 will produce first  $z$  and then, by unanimous agreement,  $y$ . In this interpretation, however, it is difficult to understand why a single person should be assigned only the choice between two selected alternatives from among the whole set, rather than the more inclusive choice among all of the alternatives. In the latter case, Person 1, given the right to choose, would select  $w$  directly; Person 2, given the right to choose, would select  $y$  directly. Hence, the assignment of decisiveness to either person would necessarily produce a position on the Pareto-optimality surface.

In his criticism of Sen's analysis, Peter Bernholz argued that under the "rule of liberalism", individuals do not have the right to choose among social states (Bernholz 1974, 100). As I shall indicate below, Bernholz's claim is correct under a reformulated and internally consistent rule of liberalism. However, Sen's statement of the rule, his Condition  $L$  or  $L^*$ , does quite explicitly grant to the individual a right to choose between a pair of social states, defined on his own terms. In response to Bernholz, Sen could suggest that, as a normative principle, liberalism is indeed concerned with such a right (Sen 1975, 112). Furthermore, he could respond that: "Given the rest of the world, Jack's choice over the 'measure' of sleeping on his belly *is* a choice over two 'social states'." (1976, 228; italics in original) Indeed it is. But no such choice could simultaneously be allowed to Jill, whose sleeping position must be fixed in Jack's 'rest of the world'.

Seidl's distinction between technologically separable and nonseparable social states also involves an implicit and indirect recognition of the contradiction (Seidl 1975, 262ff.). But, as with Bernholz, Sen could, and did, respond by pointing out that "the result of defining liberalism in the existential form is that, if it is denied, then  $j$  loses his right to decide singlehanded on the 'regime' . . . *as well* as his right to sleep as he likes . . ." (1976, 228; italics in original). By existential form, Sen here refers to his own Condition  $L$  (or  $L^*$ ), as defined. My point may be restated in this context as follows: If  $j$  is indeed given the right to decide singlehanded on the regime *as well as on everything else*,  $i$  cannot possibly be granted a similar right.

But Sen surely does not mean what he appears to say in parts of his formal proof and in some of his responses to critics. To determine just what he has in mind we may refer to his examples. In these it seems clear that he is not implicitly defining the choice alternatives for an individual as complete descriptions of society. In terms of our own example above, Sen would want to define the possible alternatives for Person 1 as the two states of his own face, independently of the state of Person 2's face. Similarly, for Person 2. Complete descriptions of the possible social states embody two separate components, and it is mutually consistent to assign each of the two persons control over one of these two. What we have here is a two-person interaction, which may be readily illustrated in simple matrix form, as in Figure 1 below.

Figure 1

		Person 2	
		<i>Clean-Shaven</i>	<i>Bearded</i>
		$z'$	$w'$
Person 1	<i>Clean-Shaven</i>	$(x'z')$ $a, c$	$(x'w')$ $b, c'$
	<i>Bearded</i>	$(y'z')$ $a', d$	$(y'w')$ $b', d'$
	$x'$		
	$y'$		

We may now define the social states by their two components, as shown in the separate cells of the matrix. We may use  $x'$  and  $y'$  to describe the pair of alternatives over which Person 1 may be allowed control, and  $z'$  and  $w'$  to describe the pair of alternatives over which Person 2 may be allowed control. (The primes are added here to suggest that these alternatives are *not* the same as those denoted by  $x, y, z$  and  $w$  above.) We can now define the four possible social states as  $x'z', x'w', y'z'$  and  $y'w'$ . In this setting, which seems to be

in conformity with the discussion in Sen's examples, the individual rankings provided in parts of his alleged proof may be quite differently interpreted. We can interpret Person 1's preference of  $x'$  over  $y'$  to mean that, given any state of the environment (in this case Person 2's behavior), Person 1 will prefer to be clean-shaven to wearing a beard. Similarly for Person 2. There is row dominance for Person 1, column dominance for Person 2. In terms of the ordinal utility payoffs represented in the matrix illustration, we know only that, for Person 1,  $a$  exceeds  $a'$  and  $b$  exceeds  $b'$ , while, for Person 2,  $c$  exceeds  $c'$  and  $d$  exceeds  $d'$ .

In this setting, it is possible to introduce a meaningful statement of the libertarian rule or principle. This would suggest that each person should be allowed control over a particular element—in our illustration, over the state of his own face. Hence, Person 1 should be allowed to choose between  $x'$  and  $y'$ , while Person 2 should be allowed to choose between  $z'$  and  $w'$ .<sup>2</sup> The solution that would emerge from the operation of this rule is  $x'z'$ . From a knowledge of the rankings of the ordinal payoffs, as interpreted above, we know *nothing* about whether this solution or outcome is or is not Pareto-dominated by some other social state. In the relationships between payoffs noted,  $b'$  may be less than, equal to or greater than  $a$ , while  $d'$  may be less than, equal to or greater than  $c$ . If  $b'$  is greater than  $a$ , and  $d'$  is greater than  $c$ , we then have the familiar prisoners' dilemma setting, in which both parties can reach agreement on a shift away from the wholly independent adjustment outcome toward the jointly efficient solution to the interaction. There is surely nothing at all new or "deeply illiberal" (Sen 1970, 157) in this.

Both to him and to most of his critics, Sen's results seem more significant than they are because of the fundamental confusion between the evaluation of complete social states and the evaluation of elements within these states, and because in switching between his examples and his formal proof, Sen mixes these two quite different conceptions. As a consequence, Sen's overall argument seems to imply a more pervasive inconsistency between Pareto-optimality and the rule of liberalism, properly reformulated, than occurs in the familiar externality analysis.

This may be demonstrated readily by looking directly at Sen's initial example in the interaction setting introduced above. Let us replace the four states of society depicted in Figure 1 by the following:

- I. Person *A* reads *Lady Chatterley's Lover* (*LCL*); Person *B* reads *LCL*.
- II. Person *A* does not read *LCL*; Person *B* reads *LCL*.
- III. Person *A* reads *LCL*; Person *B* does not read *LCL*.
- IV. Person *A* does not read *LCL*; Person *B* does not read *LCL*.

---

<sup>2</sup> This meaning of the principle is clear in Sen's later reformulation. See his "Acceptance of Personal Liberty" in 1976, 217.

In Figure 2 we may put this in matrix form analogous to that of Figure 1:

*Figure 2*

		Person B	
		<i>Reads</i>	<i>Does Not Read</i>
		<i>LCL</i>	<i>LCL</i>
Person A	<i>Reads</i>	I $a, c$	II $b, c'$
	<i>Does Not Read</i>	III $a', d$	IV $b', d'$
	<i>LCL</i>		

Sen's discussion leaves out the social state depicted in Cell I, but its inclusion does not violate the spirit of his example. He then tells us that *A*, the prude, ranks outcome IV above III, which in turn is valued more highly than II. We may add I as *A*'s lowest ranked outcome. Hence, from Sen we get the payoffs for *A* arrayed as follows:  $b' > b > a' > a$ . For Person *B*, the lascivious, adding I, we get:  $c > c' > d > d'$ . Note that these are ordinally arrayed payoffs over the full set of *complete* social states. In this they differ from the orderings in the discussion of Figure 1, which were only over pairs of the four complete states. In the orderings discussed in Figure 1, there was no way of comparing the *as* and *bs* for Person 1, or the *cs* and *ds* for Person 2. Application of the libertarian rule, therefore, might or might not produce a Pareto-optimal outcome or solution. In Sen's example, however, his explicit ranking of social states rather than the mutually-compatible, individually-controllable elements of these states, guarantees that the independent adjustment outcome will be nonoptimal. Note that, in Figure 2, if Person *A* is allowed to select his own reading matter, and if Person *B* is allowed to do the same, the outcome in Cell II is produced. This libertarian outcome is Pareto-dominated by that in Cell III by Sen's rankings. By comparison, there is no way for us to know, without additional information, that the  $x'z'$  outcome in Figure 1 was similarly Pareto-dominated.

Nowhere in his analysis, either in his original note, his several replies to critics, or in his 1976 elaboration, does Sen refer to the assignment or starting-point problem. If the purpose is limited to that of deriving an ordering over complete social states, there is no issue of assigning or partitioning rights among separate persons. But if individuals are to be allowed to interact, one with another, to produce social outcomes, their separate assignments in the interaction process must first be specified. It is at this point that the

rule or principle of liberalism or libertarianism, as reformulated in this note and earlier as interpreted by Bernholz, may be sharply distinguished from its alternatives. Sen is concerned about the presence and potential influence of 'meddlesome preferences' in social outcomes. The rule of liberalism assigns to each person a protected domain or sphere of private action that he can, if he desires, enforce as a part of any social outcome. At this level, the rule of liberalism may be contrasted with that of extreme collectivism, where no such protected domains for individual action exist.

As it is normally invoked, the Pareto principle does not enter into the analysis of initial assignments, where pure conflict is presumed to prevail.<sup>3</sup> This principle or norm emerges only at a post-assignment or post-constitutional level of analysis. Once individual rights have been assigned or partitioned, the Pareto criterion does offer a means of evaluating potential transfers of rights among individuals. At this point, 'meddlesome preferences' may reenter. If a person is assigned the right to determine his own reading matter, he can guarantee the enforcement of this right as a part of the observed social outcome. If, however, someone else places a higher value on this person's reading habits than he does himself, the Pareto norm would suggest the mutuality of gains from a transfer. In the end, the 'meddlesome preferences' may prevail, but only if those who hold them are willing to pay for their exercise.

The ambiguity that I have identified in this note seems to lie at the base of Sen's antipathy to the Pareto principle or norm. If 'rights' are somehow interpreted to allow persons to choose among complete social states, it follows from the earlier analysis here that mutually-advantageous exchanges of rights are not possible. Since only one person can choose among complete social states, no one else possesses any 'choices' that he might subsequently 'trade' or 'exchange'. In such a setting, the Pareto-superiority of alternative social states must seem "ill-defined" (Sen 1975, 112). This peculiarity disappears immediately, however, once we recognize that the rule of liberalism does not, and, indeed, cannot assign rights to choose among complete social states to anyone. Persons are assigned rights to control defined elements which, when combined with the exercise of mutually-compatible rights of others, will generate a social state as an outcome of an interaction process, not of a 'choice', as such, by either one or many persons. In this context, 'rights' may be exchanged among persons, and it is precisely in the evaluation of potential exchanges, be these simple or complex, that the Pareto criterion comes into use.

---

<sup>3</sup> In my book *The Limits of Liberty* (1975), I extend the Pareto norm backward so to speak to explain conceptually the leap from Hobbesian anarchy. But this is by no means the orthodox or standard usage of the Pareto construction.

## **Bibliography**

- Bernholz, P. (1974), Is a Paretian Liberal Really Impossible?, in: *Public Choice* 20, 99–107
- Buchanan, J. M. (1975), *The Limits of Liberty*, Chicago
- Sen, A. (1970), The Impossibility of a Paretian Liberal, in: *Journal of Political Economy* 78, 152–57
- (1975), Is a Paretian Liberal Really Impossible? A Reply, in: *Public Choice* 30, 112
- (1976), Liberty, Unanimity, and Rights, in: *Economica* 430, 217–46
- Seidl, Ch. (1975), On Liberal Values, in: *Journal of Economics/Zeitschrift für Nationalökonomie* 350, 257–92